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BACKORDER ESTIMATION UNDER MULTIPLE FAILURES OF LOWER INDENTURE ITEMS: A TECHNICAL NOTE

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Most multiechelon, multiindenture stockage models used by the Military Services and industry are extensions of the basic Multi-Echelon Technique for Recoverable Item Control (METRIC) model. Those models assume that the failure of an item is due to the failure of one and only one next lower indenture item, although more than one lower indenture item failure is observed in many real-world situations. Under such assumptions, the METRIC theory overstates backorders, often dramatically, and that overstatement results in a misallocation of spares budgets.

The author

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Executive Summary

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Most multi-echelon, multi-indenture stockage models used by the Military Services and industry are extensions of the basic Multi-Echelon Technique for Recoverable Item Control (METRIC) model. Those models assume that the failure of an item is due to the failure of one and only one next lower indenture item, although more than one lower indenture item failure is observed in many real-world situations. Under such assumptions, the METRIC theory overstates backorders, often dramatically, and that overstatement results in a misallocation of spares budgets.

We consider two types of failure detection mode for lower indenture items: simultaneous and sequential. For both problems, we developed mathematical models that produce good lower and upper bounds on the true solutions. Interpolation formulas based on univariate regression provide an estimate of the true solution. In the simultaneous detection problem, METRIC's more than 300 percent average absolute error has been reduced to less than 4 percent on a sample of 120 simulation cases; in the sequential problem, the error is reduced from 9 percent to 6 percent. The maximum errors are reduced more dramatically. The new analytic procedures are easy to calculate, and they should be relatively easy to incorporate into computer programs for spares optimization.

CONTENTS

CHAPTER 1

INTRODUCTION

OBJECTIVE

The objective of this research is to find easily computed approximations to the expected backorders (EBOs) when multiple failures of lower indenture items can occur. The average of the absolute percent error over a large number of cases should be low, but the maximum percent error should also be acceptable.

RATIONALE

Approximation techniques for the multiple failure problem are valuable for three major reasons:

- When an item is repaired, more than one next lower indenture item is often also repaired or replaced.
- Current multi-indenture stockage models in the Multi-Echelon Technique for Recoverable Item Control (METRIC) [1] family of models dramatically overstate backorders when multiple failures occur; errors in excess of 100 percent are common. This overstating helps to explain why the models almost always predict availabilities lower than those actually achieved in the field.
- The assumption of one and only one lower indenture failure is the reason that a simple analytic solution for multiple failures is obtained in multi-indenture METRIC models. The analytic solution for multiple failures is extremely complicated because the lower indenture item backorder computations are no longer independent.

PROBLEM DESCRIPTION

Our description is presented in terms of two indentures: a first-indenture, line replaceable unit (LRU) and its second-indenture, shop replaceable units (SRUs) at one site. That description simplifies the discussion, and the results can be extended to more indentures and echelons.

The SRU failure detection process can proceed in two very different ways: simultaneously and sequentially. In the simultaneous case, we assume that after

some LRU checkout time, a diagnosis of all failed SRUs is made at a point in time. In the sequential case, we assume that after some LRU checkout time, a diagnosis of the first failed SRU is made. If a spare SRU is available, it is installed on the LRU and the testing continues to find the next (if any) failed SRU. However, if a spare SRU is not available, the diagnosis of the next failed SRU is delayed.

With sequential detection, the EBOs are much larger than in the case of simultaneous detection. The natural question is which process is closer to reality?

We describe the Air Force failure detection process for LRUs with automatic test equipment (ATE), as performed in an Aircraft Instrument Shop. The LRU is placed on a test stand, and a series of automated tests is performed until a failure is detected in some SRU. A replacement SRU is installed, and the automatic test sequence is restarted at the "break point" in the software preceding the last test that failed. A spare SRU is usually available for installation, because a mock-up or shop standard is available and good SRUs can be pulled from it. Since the detection of failed SRUs does not require that any SRU be repaired before testing can continue, the detection process is approximately *simultaneous* (the test sequence may require 24 to 36 hours, but that time is included in the model as the time for LRU checkout). The assumption is that after the LRU has been completely diagnosed, any SRUs that were taken from the shop standard – thus creating "holes" – are replaced. The LRU is not considered ready for service until the SRU holes are filled and the LRU is retested. However, the shop can begin to repair the failed SRUs immediately.

The simultaneous detection scenario is a little optimistic, of course, since it is possible that a given SRU from the mock-up will be required in two or more LRUs at the same time. There may be other reasons that the simultaneous model is too optimistic, particularly on LRUs without ATE. In some cases no mock-ups are available and thus a good SRU may not always be ready for installation. For those reasons, we model both types of detection.

MULTIPLE FAILURE DATA

Before modeling the multiple SRU failure problem, we introduce some evidence that the problem really does exist. In a recent report to the Air Force [2], we examined detailed, hierarchical data for 10 LRUs and their families of lower indenture parts (to the fifth indenture in some cases) and found that lower indenture demand was often much greater than that for the parent item. For one LRU that

ratio exceeded 10.[1] This effect becomes more important at the depot and as one moves to lower indentures.

Another source of multiple SRU failures is battle damage in wartime scenarios. It is quite likely that more than one SRU located close together will be affected. Unfortunately, we have no detailed data on this phenomenon.

REPORT ORGANIZATION

We address the simultaneous detection problem in the next section and the sequential detection problem in the following chapter. In each case, we provide background on the problem, a description of the simulation and the analytic model used to compute lower and upper bounds on the solution, the interpolation procedure suggested for estimating the solution, and the numerical results and an analysis of them. The final chapter presents our conclusions. We also provide an appendix showing how to compute Erlang state probabilities for any mean and any variance-to-mean ratio less than one.

[1]The ratios of total SRU demand divided by LRU demand for the 10 LRUs were 10.21, 5.80, 0.66, 0.08, 1.48, 0.75, 2.97, 0.55, 0.06, and 0.00; the corresponding ratios for the depot replacement factors [including second-indenture economic order quantity (EOQ) items as well as reparables] were 240.94, 4.43, 0.44, 2.31, 23.10, 0.85, 14.97, 136.53, 0.34, and 2.31. The latter set of numbers should be higher, but we know from an examination of the Illustrated Parts Breakdown that it is overstated in some cases because of errors in the parts hierarchy data. In particular, the value of 240.94 was found to be much too large because many of the EOQ items listed as second-indenture are really lower indenture parts.

CHAPTER 2

SIMULTANEOUS FAILURE DETECTION

BACKGROUND

Let N denote the number of SRUs on the LRU. When the LRU fails, there is a probability p(i), where $0 \leq p(i) \leq 1$ and $i = 1, 2, \ldots N$, for the failure of each SRU. In other words, from 0 to N failures of SRUs may occur whenever an LRU fails. We assume that the SRU failures are independent of each other (that assumption is probably not strictly valid, but no data are available to support more complicated assumptions).

In the simpler, single SRU failure model, the LRU Poisson demand process splits neatly into N independent SRU Poisson processes (assuming that the p(i)'s sum to one or less). Since there is only one SRU in each process, the EBOs can be computed independently. Furthermore, under the ample service assumption, Palm's theorem can be invoked; it states that the EBOs are independent of the shape of the repair distribution [3].

In the multiple failure case, the LRU Poisson demand process splits into 2^N independent Poisson processes (each of the N SRUs may be in one of two states – good or bad). For a particular SRU, Palm's theorem still applies. However, since several SRUs may have failed simultaneously, the backorder computations for each SRU are not independent. Suppose no spare SRUs are available when the LRU fails. The LRU repair cannot be completed until all failed SRUs have been repaired. This implies that the waiting time and the EBOs for the LRU will be longer when the SRU repair times are more variable (Palm's theorem does not apply to the group of SRUs). Since the results depend on the shape of the repair distribution for each SRU, we must decide whether the variability extremes of constant or exponential repair distributions are reasonable or some intermediate distribution is preferable.

The 2^N failure processes for the new model are much larger than N (e.g., when $N = 5$, $2^N = 32$) and grow very rapidly. The computation of backorders is further

complicated by the fact that stock is sometimes available for one or more of the failed SRUs.

In an earlier report for the Air Force [4], we developed methods for modeling the impact of multiple simultaneous failures and built that capability into an evaluation model. The basic idea was to divide the repair of an LRU into a set of mutually exclusive, collectively exhaustive "processes," where a process was defined to be a set of next lower indenture SRU failures. We applied standard multi-echelon theory to compute the probability distribution for the number of units of each process in repair. From those probabilities and the number of spares on each SRU, we were then able to compute the probability distribution for the number of EBOs on each SRU. Then, assuming that any SRU EBOs are consolidated on the fewest LRUs (cannibalization), we were able to calculate the LRU EBOs and availability.

The computational procedure is straightforward, but highly time-consuming when more than four or five processes or items are involved. The ability to perform those calculations is useful in an evaluation model but would not be practical in an optimization model where it would have to be performed many times. Furthermore, the capability was implemented only for a single base with two indentures.

Another limitation of that model is that each item in a process must have the same repair time, and that repair time must be a constant. For all of those reasons, the analytic calculation used in that evaluation model is not considered further in this report.

REPAIR DISTRIBUTION

In the multiple SRU failure case, the LRU EBOs depend on the shape of the repair distribution for each SRU. Suppose for simplicity that all stock levels are zero and that each SRU has the same mean repair time of 1. If the SRU repair times are constant, the waiting time to repair the SRUs is 1 regardless of the number that failed; if they are exponential, the waiting time increases with the number of SRUs as shown in the second column of Table 2-1.

Of course, constant and exponential repair times represent two extremes of no variability and high variability, respectively. By using an Erlang distribution (the sum of k exponential variables), we can obtain results between the extremes. Results for the Erlang-2, -3, and -4 obtained by simulation are shown in Table 2-1.

TABLE 2-1

EXPECTED TIME UNTIL THE LAST OF N SRU REPAIRS IS COMPLETE, EACH WITH MEAN 1

Number of SRUs	Probability distribution			
	Exponential	Erlang–2	Erlang–3	Erlang–4
1	1.00	1.00	1.00	1.00
2	1.50	1.37	1.30	1.27
3	1.83	1.60	1.49	1.43
4	2.08	1.77	1.63	1.55
5	2.28	1.90	1.73	1.64
6	2.45	2.01	1.82	1.70
7	2.59	2.10	1.89	1.76
8	2.72	2.18	1.95	1.81
9	2.83	2.26	2.00	1.86
10	2.93	2.32	2.05	1.90

The Erlang is an appealing physical model since one can visualize repair as the sum of several independent activities, such as test, diagnosis, repair, and retest. If each activity has an exponential distribution with the same mean, the total time has an Erlang distribution. In the Erlang distribution, the value of k is usually considered to be an integer between 1 and infinity (constant repair time), but the distribution is defined for all $k > 0$ and is better known as the gamma distribution.

Table 2-1 shows that the Erlang-4 gives waiting times that are about midway between those of the exponential and the constant (whose waiting time is one). Since the shape of the repair distribution matters and raw data do not show empirical distributions for repair time that are at either extreme, we will use the Erlang-4 values as a reasonable compromise.

SIMULATION

Simulation is used to estimate the LRU EBOs under multiple SRU failures. LRUs are assumed to fail in accordance with a Poisson process. When an LRU fails, a random number is drawn for each SRU. If the random number is less than the SRU probability of failure, that SRU is deemed to have failed. In that way it is

possible for 0, 1, 2 . . . N SRUs to fail. In some cases, one or more of the SRUs in the LRU may have a quantity per next higher assembly (QPA) greater than one. Thus, more than one unit of a particular SRU may fail.

Repair times can be exponential, constant, or Erlang-k. Simulations were run for periods of 40,000 to 400,000 days, depending on demand rates, so that acceptably precise 95 percent confidence intervals around the LRU EBOs could be obtained.

Our primary interest was to estimate LRU backorders under the assumption of full cannibalization of all SRUs (i.e., consolidation of SRU shortages into the fewest possible LRUs). Although full cannibalization is not practiced unless it is necessary to achieve an availability target, we believe it provides a useful target for management concerning the highest performance level achievable.

The simulation was also run under the assumption of no cannibalization, where we assume that management keeps LRU backorders to a minimum by replacing SRUs on an LRU only when that action will make the LRU serviceable. A repaired SRU that would not restore an LRU to serviceable condition is put on the shelf until it, perhaps in combination with other SRUs, can be used to fix an LRU. It is easy to show that such a policy is better than replacing SRUs without regard to their effect on LRU condition. What is interesting is that for many combinations of stock levels, the LRU backorders under this "opportunistic" policy are not substantially greater than those under a full cannibalization policy. This is demonstrated in the examples below.

APPROXIMATE MATHEMATICAL MODEL

We consider the case of full cannibalization of SRUs belonging to a single LRU at a base. When LRU disassembly and fault isolation are completed, those SRUs that have failed are identified. If spare SRUs are available on the shelf or can be cannibalized from LRUs that are missing other SRUs, the LRU is returned to a serviceable condition. Otherwise the LRU will have to wait until spare SRUs become available to fill all of its holes.

The steady-state probability that no LRUs are waiting because of SRU shortages is the product over the SRUs of the probabilities that s_i or fewer units of SRU i are in base repair (where s_i is the base stock level for Item i); the steady-state probability that y or fewer LRUs are waiting because of SRU shortages, Q(y), is the

product over the SRUs of the probabilities that $s_i + a_i y$ or fewer units of SRU i are in base repair:

$$Q(y) = \prod_i P_i(s_i + a_i y) \qquad y = 0, 1, 2, \ldots \qquad \text{[Eq. 2-1]}$$

where $P_i(s_i + a_i y)$ is the steady-state probability of $s_i + a_i y$ or less units of the i^{th} SRU in repair, and a_i is the number of applications of SRU i in the LRU. When demand is Poisson, these state probabilities are Poisson, independent of the shape of the repair distribution.[2] However, Equation 2-1 is not strictly correct, because the SRU state probabilities are not independent of each other. This is because of the assumption that when an SRU failure is detected, there may be other SRU failures detected simultaneously.

The probability of exactly y LRUs waiting because of SRU shortages, denoted by S(y), is the probability of y or less minus the probability of y − 1 or less:

$$S(y) = Q(y) - Q(y-1) \qquad y = 1, 2, 3, \ldots \qquad \text{[Eq. 2-2]}$$

$$S(0) = Q(0)$$

The final objective is to compute the probability distribution that there are y LRU backorders because of either LRUs in disassembly and fault isolation on the one hand or LRUs whose repair is delayed by SRU shortages on the other hand. The former are Poisson probabilities (denoted by L) with mean mT_0, where m is the demand rate, T_0 is the mean time for LRU fault isolation and reassembly after SRU repair. This follows from Palm's theorem, and the observation that the two parts of the Poisson process (LRU checkout and SRU repair) are independent since they are displaced in time.

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[2]For any SRU with a QPA greater than one, the demand process is compound Poisson. In such a process, demands appear in clusters. If an individual unit of the SRU has a failure probability p and the QPA is n, the compounding distribution for the size of the cluster is binomial with mean np and variance $np(1-p)$. The extended form of Palm's theorem shows that the state probabilities are compound Poisson if the same repair time is drawn for each failed unit of the SRU cluster — a somewhat unrealistic assumption.

Even though the compounding distribution is simple, the state probabilities for the number of demands in an interval of time are not. Many references show that the mean of the compound Poisson process for the SRU is the Poisson mean multiplied by np and the variance-to-mean is the second moment of the compounding distribution divided by the first moment, or $1 + (n-1)p$. As in VARI-METRIC [3], we have adopted the simplest procedure for approximating the state probabilities by using these two parameters to define a negative binomial distribution.

If the LRU stock level is s_0, the number of backordered LRUs resulting from LRU disassembly and fault isolation, L(y), or from SRU backorders is obtained by convolution:

$$B(y) = \sum_{z=0}^{y+s_0} L(z)\, S(y+s_0-z) \qquad y = 1, 2, 3, \ldots \qquad \text{[Eq. 2-3]}$$

and the LRU EBOs are:

$$E[B(y)] = \sum_{y} yB(y). \qquad \text{[Eq. 2-4]}$$

As noted above, Equation 2-4 does not give an exact solution to our problem because the P's in Equation 2-1, although Poisson because of Palm's theorem, are not independent. If independence is assumed for computational purposes, the EBOs will be overstated. Thus, this procedure will produce an upper bound for the true solution.

We can compute a lower bound as well by using just one SRU (since the backorders with many SRUs must be at least as large). We have found that when failure probabilities or repair times vary from one SRU to another, using the average pipeline value (i.e., LRU demand rate times the average conditional probability of failure times the average SRU repair time) is appropriate, though not a strict lower bound.

In the next section (*Results*), we show that the difference between the lower and upper bound increases as the number of SRUs increases and as the sum of the SRU failure probabilities, PSUM, increases. When PSUM is 10, the simulated LRU EBOs under cannibalization, CAN, tends to be about 55 percent of the distance from the lower bound to the upper bound. As PSUM decreases, CAN is a larger percentage of the distance between the bounds. Using regression, we found that the best fit was obtained with:

$$F = 0.812 - 0.114\, \text{Log(PSUM)}, \qquad \text{[Eq. 2-5]}$$

where F is the fraction of the difference between the lower and upper bound. Since the true solution always lies in the interval, F should be constrained between 0 and 1 (we actually constrained it between 0.2 and 0.8). With 120 data points the statistical fit was extremely good and the coefficient values highly significant.

We made one alteration in the computation of upper bounds. When there are several SRUs and each has a conditional probability of failure equal to one, the upper bound is not very close to the solution, CAN. This is because the upper bound assumes independence, though in fact there is total dependence between the SRUs. Instead of computing the upper bound using all N SRUs, a better upper bound is obtained by using only N/2. This is because of our use of the Erlang-4 distribution for repair times, which, as noted in Table 2-1, gives waiting times that are about half way between the constant (single SRU) and exponential (N SRUs) cases.

RESULTS

The numerical results of applying the approximation techniques described are shown in Table 2-2. The headings for each group of cases give the number of SRUs, the daily demand rate (DDR) for the LRU, and the regression value F expressed as a percent. Next we show the conditional failure probability for each SRU (i.e., given that the LRU has just failed, the probability that a particular SRU has failed), and the average repair time for the LRU and each SRU. The repair distribution for each item is assumed to be Erlang-4. PSUM, the sum of the SRU failure probabilities, is also shown because it is the independent variable in the regression adjustment.

Each line in the table provides the results for one case:

- *Case #:* Case number for reference.
- *ALOW:* Analytic lower bound for the LRU EBOs from the model.
- *AUP:* Analytic upper bound for the LRU EBOs from the model.
- *EST:* Estimated solution for the LRU EBOs obtained by using the regression formula for the fraction of the distance between ALOW and AUP.
- *CAN:* Simulation result for the LRU EBOs under cannibalization.
- *% ERR:* The percent error which is 100(EST – CAN)/CAN.
- *DELTA:* The delta value to be added and subtracted to CAN to obtain the 95 percent confidence interval for the LRU EBOs from the simulation. If EST falls outside the confidence limits for CAN, an asterisk is placed after the % ERR to indicate a statistically significant difference.
- *MLOW:* The METRIC lower bound estimate of LRU EBOs. In those cases for which PSUM is more than one, the SRU failure probabilities are normalized so that they add to one.

- *MUP:* The METRIC upper bound estimate of LRU EBOs, without normalizing the SRU failure probabilities. Note that when PSUM is one or less, MUP = MLOW.

- *NOCAN:* Simulation result for the LRU EBOs with no cannibalization, but with the opportunistic replacement policy described in the *Simulation* section. Note that these values equal or exceed CAN.

- *Stock Levels:* Stock levels for the LRU and each SRU.

The cases in Table 2-2 are in ascending order of PSUM, the sum of the SRU failure probabilities, except that the final cases, 116 through 120, have multiple applications (QPA) of some SRUs in the LRU.

Before discussing the results, some general comments on the evaluation procedures are in order. Our primary interest is the average of the absolute values of the percent errors over the cases. This is because an error of 0.1 backorders is more important if the true value is 0.2 than if the true value is 20.

The METRIC estimate was obtained by averaging the upper- and lower-bound estimates. This was the best METRIC procedure we could find, although it is clearly not very good for several reasons:

- When the SRU failure probabilities sum to one or less, the lower and upper bounds are identical. Thus, they do not bound the solution.

- When the SRU failure probabilities sum to more than one so that the bounds are different, they are often very far apart. Even though these bounds are far apart, they fail to include the solution in 10 cases.

Though it may be possible to obtain a better estimate from METRIC, we did not attempt to do so because the recommended procedure is so much better. In that sense, METRIC is used only as a strawman for comparison purposes. That is why we show only the average METRIC error rather than the METRIC error for each individual case. The METRIC error is relevant, though, because METRIC is used to model applications now.

The cases in Table 2-2 were chosen to provide an interesting range of situations with 1 to 20 SRUs, PSUMs from 0.1 to 10, demand rates of 0.1/day and 1/day, and SRU repair times of 5 to 20 days. The average absolute percentage error depends on the mix of cases chosen and should only be considered as an indication of the adequacy of our approach.

TABLE 2-2

SIMULTANEOUS DETECTION CASES

Number of SRUs: 1 | **LRU DDR:** 0.5 | **F:** 80.0 | **Sum of SRU Failure Probabilities:** PSUM = 0.1
Conditional SRU Failure Probabilities: 0.1 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
1	1.681	1.681	1.681	1.676	0.30	0.023	1.680	1.680	1.676	1	1
2	1.597	1.597	1.597	1.600	− 0.19	0.025	1.597	1.597	1.600	1	2
3	0.949	0.949	0.949	0.947	0.21	0.018	0.946	0.946	0.947	2	1

Number of SRUs: 3 | **LRU DDR:** 0.5 | **F:** 80.0 | **Sum of SRU Failure Probabilities:** PSUM = 0.6
Conditional SRU$_i$ Failure Probabilities: 0.1 0.2 0.3
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
4	1.928	2.538	2.416	2.453	− 1.51	0.038	2.722	2.722	2.538	1	1 1 1
5	1.155	1.388	1.341	1.327	1.09	0.031	1.458	1.458	1.370	2	1 1 2

Number of SRUs: 1 | **LRU DDR:** 0.5 | **F:** 80.0 | **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 1.0 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
6	1.002	1.002	1.002	0.987	1.52	0.040	0.905	0.905	0.987	3	5

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 2 **LRU DDR:** 0.02 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
7	0.090	0.138	0.128	0.121	5.85*	0.001	0.140	0.140	0.122	0	0 0
8	0.004	0.007	0.006	0.006	4.59*	0.000	0.009	0.009	0.007	1	0 0

Number of SRUs: 2 **LRU DDR:** 0.1 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
9	0.450	0.650	0.610	0.589	3.57*	0.008	0.700	0.700	0.598	0	0 0
10	0.088	0.147	0.135	0.133	1.59	0.003	0.197	0.197	0.144	1	0 0
11	0.026	0.033	0.031	0.032	– 1.75	0.001	0.031	0.031	0.032	1	1 1

Number of SRUs: 2 **LRU DDR:** 0.3 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
12	1.350	1.790	1.702	1.668	2.04*	0.025	2.100	2.100	1.748	0	0 0
13	0.609	0.912	0.851	0.828	2.83*	0.019	1.222	1.222	0.911	1	0 0
14	0.276	0.388	0.366	0.365	0.16	0.008	0.397	0.397	0.365	1	1 1

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 2 **LRU DDR:** 0.5 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
15	2.250	2.845	2.726	2.695	1.15	0.034	3.500	3.500	2.811	0	0 0

Number of SRUs: 2 **LRU DDR:** 1.0 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
16	4.500	5.369	5.195	5.161	0.66	0.079	7.000	7.000	5.374	0	0 0
17	3.510	4.369	4.197	4.206	− 0.21	0.026	6.000	6.000	4.376	1	0 0
18	0.620	0.987	0.914	0.914	− 0.04	0.016	2.293	2.293	1.044	5	0 0
19	0.841	1.094	1.043	1.052	− 0.82	0.018	1.112	1.112	1.064	2	3 3

Number of SRUs: 2 **LRU DDR:** 5.0 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
20	22.500	24.484	24.087	23.803	- 1.19	0.790	35.000	35.000	24.425	0	0 0
21	0.923	1.518	1.399	1.405	− 0.43	0.036	10.082	10.082	1.569	25	0 0

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 2 **LRU DDR:** 20.0 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
22	90.000	93.984	93.187	93.066	0.13	1.482	140.000	140.000	93.474	0	0 0

Number of SRUs: 3 **LRU DDR:** 0.3 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU_i Failure Probabilities: 0.60 0.30 0.10
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
23	1.100	1.761	1.629	1.661	− 1.94*	0.020	2.100	2.100	1.748	0	0 0 0
24	0.433	0.883	0.793	0.825	− 3.88*	0.018	1.222	1.222	0.911	1	0 0 0
25	0.132	0.345	0.302	0.327	− 7.52*	0.012	0.602	0.602	0.385	2	0 0 0
26	1.100	1.347	1.298	1.263	2.74*	0.020	1.507	1.507	1.317	0	1 0 0
27	0.433	0.580	0.551	0.536	2.72	0.015	0.728	0.728	0.586	1	1 0 0
28	0.132	0.190	0.178	0.178	0.22	0.008	0.284	0.284	0.208	2	1 0 0
29	0.707	0.984	0.929	0.957	− 2.97*	0.017	1.005	1.005	0.958	0	1 1 1
30	0.205	0.372	0.339	0.363	− 6.72*	0.012	0.371	0.371	0.362	1	1 1 1
31	0.046	0.113	0.100	0.113	−11.86*	0.006	0.105	0.105	0.112	2	1 1 1

Number of SRUs: 10 **LRU DDR:** 0.3 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.1 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
32	0.154	0.198	0.189	0.192	− 1.46	0.004	0.200	0.200	0.196	1	1 1 1 1 1 1 1 1 1 1

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 3 **LRU DDR:** 0.1 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU_i Failure Probabilities: 0.3 0.4 0.3
Repair Times: 5 for LRU; SRU_i = 40 40 20

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
33	0.829	2.243	1.960	2.135	− 8.19*	0.026	3.102	3.102	2.378	0	0 1 0
34	0.118	0.224	0.203	0.214	− 5.23*	0.010	0.732	0.732	0.344	3	0 1 0

Number of SRUs: 3 **LRU DDR:** 1.0 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU_i Failure Probabilities: 0.3 0.4 0.3
Repair Times: 5 for LRU; SRU_i = 40 40 20

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
35	13.340	19.940	18.620	17.551	. 6.09*	0.225	35.000	35.000	19.150	3	0 1 0
36	3.005	3.785	3.629	3.696	− 1.81	0.078	6.679	6.679	4.009	10	12 8 4
37	0.355	0.401	0.391	0.398	− 1.64	0.016	13.024	13.024	0.754	25	0 1 0

Number of SRUs: 5 **LRU DDR:** 0.1 **F:** 78.66 **Sum of SRU Failure Probabilities:** PSUM = 1.25
Conditional SRU Failure Probabilities: 0.25 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
38	0.158	0.222	0.208	0.195	6.84*	0.005	0.126	0.243	0.195	1	1 1 1 1 1

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 3 **LRU DDR:** 0.1 **F:** 75.15 **Sum of SRU Failure Probabilities:** PSUM = 1.7

Conditional SRU_i Failure Probabilities: 0.6 0.8 0.3

Repair Times: 5 for LRU; 20 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
39	1.634	2.642	2.392	2.381	0.44	0.024	2.500	3.900	2.471	0	0 0 0
40	0.829	1.662	1.455	1.443	0.83	0.023	1.582	2.920	1.534	1	0 0 0
41	0.341	0.858	0.730	0.728	0.21	0.017	0.869	2.019	0.807	2	0 0 0
42	1.634	2.243	2.092	2.004	4.37*	0.022	1.890	3.102	2.088	0	0 1 0
43	0.829	.2961	1.180	1.111	6.21*	0.020	1.041	2.147	1.198	1	0 1 0
44	0.341	0.599	0.535	0.503	6.34*	0.013	0.478	1.331	0.573	2	0 1 0
45	0.118	0.223	0.197	0.189	4.18	0.008	0.184	0.732	0.228	3	0 1 0
46	0.956	1.676	1.497	1.483	0.95	0.022	1.087	1.952	1.497	0	1 1 1
47	0.372	0.861	0.739	0.737	0.34	0.017	0.424	1.094	0.754	1	1 1 1
48	0.122	0.362	0.302	0.308	- 1.83	0.011	0.128	0.513	0.320	2	1 1 1
49	0.035	0.127	0.104	0.111	- 6.18*	0.005	0.031	0.203	0.115	3	1 1 1
50	0.642	0.981	0.897	0.902	- 0.58	0.016	0.637	1.018	0.905	0	2 2 2
51	0.184	0.389	0.338	0.346	- 2.30	0.011	0.166	0.379	0.347	1	2 2 2
52	0.535	0.654	0.624	0.629	- 0.73	0.013	0.526	0.657	0.631	0	3 3 3
53	0.125	0.192	0.175	0.182	- 3.55*	0.006	0.117	0.176	0.182	1	3 3 3

Number of SRUs: 3 **LRU DDR:** 1.0 **F:** 75.15 **Sum of SRU Failure Probabilities:** PSUM = 1.7

Conditional SRU_i Failure Probabilities: 0.6 0.8 0.3

Repair Times: 5 for LRU; 20 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
54	16.340	21.682	20.355	21.088	- 3.48*	0.306	25.000	39.000	21.397	0	0 0 0
55	14.340	19.682	18.355	19.090	- 3.85*	0.296	24.000	37.000	19.399	2	0 0 0
56	15.340	20.622	19.309	20.089	- 3.88*	0.301	22.031	36.003	20.345	0	1 1 1
57	0.355	0.401	0.390	0.391	- 0.37	0.034	1.501	13.024	0.438	25	0 1 0

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 5 **LRU DDR:** 0.1 **F:** 70.75 ***Sum of SRU Failure Probabilities:*** PSUM = 2.5
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
58	0.032	0.088	0.071	0.071	0.10	0.003	0.026	0.112	0.079	2	1 1 1 1 1

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 62.85 ***Sum of SRU Failure Probabilities:*** PSUM = 5.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
59	1.000	2.259	1.791	1.767	1.38*	0.016	2.500	5.500	1.988	0	0 0 0 0 0 0 0 0 0 0
60	0.368	1.263	0.931	0.939	- 0.90	0.013	0.723	4.504	1.106	1	0 0 0 0 0 0 0 0 0 0
61	1.000	1.957	1.601	1.580	1.36*	0.015	1.024	3.533	1.726	0	0 1 0 1 0 1 0 1 0 1
62	0.368	0.989	0.758	0.736	3.03*	0.012	0.383	2.562	0.884	1	0 1 0 1 0 1 0 1 0 1
63	0.607	1.266	1.021	0.964	5.93*	0.013	0.549	1.565	1.001	0	1 1 1 1 1 1 1 1 1 1
64	0.032	0.144	0.102	0.100	2.39	0.004	0.021	0.311	0.118	2	1 1 1 1 1 1 1 1 1 1
65	0.502	0.519	0.513	0.518	- 0.99	0.006	0.500	0.519	0.518	0	3 3 3 3 3 3 3 3 3 3
66	0.107	0.115	0.112	0.114	- 1.38	0.003	0.107	0.114	0.114	1	3 3 3 3 3 3 3 3 3 3
67	0.158	0.337	0.271	0.277	- 2.34	0.007	0.116	0.395	0.277	1	3 3 3 3 3 1 1 1 1 1
68	0.114	0.179	0.155	0.157	- 1.24	0.004	0.107	0.178	0.159	1	2 2 2 2 2 2 2 2 2 2

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 62.85 **Sum of SRU Failure Probabilities:** PSUM = 5.0

Conditional SRU_i Failure Probabilities: 0.2 0.3 0.4 0.4 0.5 0.5 0.6 0.6 0.7 0.8

Repair Times: 5 for LRU; SRU_i = 5 10 5 10 5 10 5 10 5 10

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
69	0.875	2.092	1.640	1.634	0.36	0.017	1.260	4.300	1.767	0	0 0 0 0 0 0 0 0 0 0
70	0.292	1.105	0.803	0.789	1.77*	0.013	0.544	3.314	0.922	1	0 0 0 0 0 0 0 0 0 0
71	0.875	1.564	1.308	1.171	11.70*	0.013	0.769	2.316	1.229	0	0 1 0 1 0 1 0 1 0 1
72	0.562	1.114	0.909	0.891	2.01*	0.013	0.535	1.264	0.917	0	1 1 1 1 1 1 1 1 1 1
73	0.136	0.423	0.316	0.315	0.44	0.008	0.121	0.546	0.336	1	1 1 1 1 1 1 1 1 1 1
74	0.024	0.120	0.084	0.088	- 4.16	0.004	0.020	0.186	0.100	2	1 1 1 1 1 1 1 1 1 1

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 62.85 **Sum of SRU Failure Probabilities:** PSUM = 5.0

Conditional SRU_i Failure Probabilities: 0.2 0.3 0.4 0.4 0.5 0.5 0.6 0.6 0.7 0.8

Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels LRU	Stock Levels SRUs
75	1.000	2.325	1.833	1.794	2.16*	0.017	1.500	5.500	1.973	0	0 0 0 0 0 0 0 0 0 0
76	0.368	1.329	0.972	0.914	6.35*	0.015	0.723	4.504	1.094	1	0 0 0 0 0 0 0 0 0 0
77	1.000	1.991	1.623	1.584	2.45*	0.015	1.009	3.516	1.711	0	0 1 0 1 0 1 0 1 0 1
78	0.368	1.023	0.780	0.749	4.10*	0.013	0.373	2.546	0.875	1	0 1 0 1 0 1 0 1 0 1
79	0.104	0.381	0.278	0.269	3.38*	0.007	0.106	1.680	0.359	2	0 1 0 1 0 1 0 1 0 1
80	0.607	1.332	1.063	0.994	6.91*	0.014	0.554	1.657	1.029	0	1 1 1 1 1 1 1 1 1 1
81	0.032	0.169	0.118	0.112	5.45*	0.005	0.022	0.354	0.131	2	1 1 1 1 1 1 1 1 1 1

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 62.85 **Sum of SRU Failure Probabilities:** PSUM = 5.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 20 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels: LRU	Stock Levels: SRUs
82	0.723	2.238	1.675	1.719	− 2.55*	0.023	1.582	9.500	2.249	1	0 0 0 0 0 0 0 0 0 0
83	0.723	1.893	1.458	1.463	− 0.32	0.020	0.797	6.340	1.840	1	0 1 0 1 0 1 0 1 0 1
84	0.314	1.266	0.912	0.900	1.37	0.018	0.190	3.205	1.037	1	1 1 1 1 1 1 1 1 1 1
85	0.095	0.543	0.377	0.373	0.96	0.011	0.039	2.273	0.485	2	1 1 1 1 1 1 1 1 1 1

Number of SRUs: 10 **LRU DDR:** 1.0 **F:** 62.85 **Sum of SRU Failure Probabilities:** PSUM = 5.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 20 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels: LRU	Stock Levels: SRUs
86	5.137	10.118	8.268	8.915	− 7.26*	0.082	15.000	95.000	12.725	10	0 0 0 0 0 0 0 0 0 0
87	0.221	1.288	0.892	0.925	− 3.61	0.044	0.022	7.546	1.205	10	10 10 10 10 10 10 10 10 10 10

Number of SRUs: 6 **LRU DDR:** 0.1 **F:** 60.77 **Sum of SRU Failure Probabilities:** PSUM = 6.0
Conditional SRU Failure Probabilities: 1.0 for each SRU
Repair Times: 5 for LRU; 20 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels: LRU	Stock Levels: SRUs
88	1.041	1.778	1.489	1.547	− 3.76*	0.027	0.331	3.748	1.547	0	2 2 2 2 2 2
89	0.882	1.750	1.410	1.475	− 4.44*	0.027	0.249	6.313	1.475	1	1 1 1 1 1 1
90	0.869	1.750	1.404	1.472	− 4.59*	0.027	0.870	10.500	1.472	2	0 0 0 0 0 0

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 1 0 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
91	1.500	2.732	2.177	2.136	1.92*	0.022	1.500	10.500	2.136	0	0 0 0 0 0 0 0 0 0 0
92	0.723	1.737	1.280	1.229	4.17*	0.019	0.723	9.500	1.229	1	0 0 0 0 0 0 0 0 0 0
93	0.281	0.870	0.605	0.576	4.98*	0.014	0.281	8.500	0.576	2	0 0 0 0 0 0 0 0 0 0
94	0.723	1.481	1.140	1.086	4.93*	0.014	0.383	6.340	1.086	1	0 1 0 1 0 1 0 1 0 1
95	0.313	0.870	0.619	0.590	4.93*	0.014	0.126	3.205	0.590	1	1 1 1 1 1 1 1 1 1 1
96	0.095	0.335	0.227	0.222	2.20	0.008	0.021	2.273	0.222	2	1 1 1 1 1 1 1 1 1 1
97	0.162	0.354	0.268	0.270	− 0.92	0.009	0.107	0.752	0.270	1	2 2 2 2 2 2 2 2 2 2
98	0.523	0.613	0.572	0.576	− 0.62	0.009	0.500	0.734	0.576	0	3 3 3 3 3 3 3 3 3 3

Number of SRUs: 10 **LRU DDR:** 1.0 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 1.0 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
99	0.221	0.803	0.541	0.605	− 10.61*	0.025	0.022	7.546	0.605	10	10 10 10 10 10 10 10 10 10 10
100	0.013	0.059	0.038	0.047	− 17.86*	0.006	0.000	3.175	0.047	15	10 10 10 10 10 10 10 10 10 10

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels: LRU	Stock Levels: SRUs
101	1.000	2.639	1.901	1.987	− 4.35*	−0.044	1.500	10.500	2.201	0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
102	0.368	1.639	1.066	1.021	4.45*	0.014	0.723	9.500	1.295	1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
103	0.104	0.730	0.448	0.410	9.27*	0.009	0.281	8.500	0.630	2	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
104	0.023	0.230	0.137	0.125	9.40*	0.005	0.090	7.502	0.249	3	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
105	1.000	2.260	1.692	1.749	− 3.24*	0.016	1.000	5.519	1.958	0	0 3 0 3 0 3 0 3 0 3 0 3 0 3 0 3 0 3 0 3
106	0.502	0.538	0.522	0.522	− 0.04	0.006	0.500	0.539	0.523	0	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
107	0.263	1.339	0.854	0.880	− 2.92*	0.013	0.376	5.567	1.088	1	0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
108	0.158	0.730	0.472	0.423	11.66*	0.010	0.116	1.703	0.465	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
109	0.032	0.230	0.141	0.127	10.87*	0.005	0.019	0.964	0.156	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
110	0.114	0.244	0.185	0.178	4.18*	0.005	0.107	0.264	0.182	1	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
111	0.158	0.506	0.349	0.343	1.75	0.008	0.111	0.790	0.369	1	3 3 3 3 3 3 3 3 3 3 1 1 1 1 1 1 1 1 1 1

Number of SRUs: 20 **LRU DDR:** 1.0 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels: LRU	Stock Levels: SRUs
112	0.003	0.040	0.023	0.028	− 16.17	0.007	0.000	7.628	0.067	15	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
113	0.263	0.391	0.333	0.342	− 2.53	0.016	0.255	0.376	0.347	7	10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU$_i$ Failure Probabilities: 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8
Repair Times: 5 for LRU; 10 for each SRU

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
114	0.502	0.604	0.558	0.557	0.19	0.008	0.500	0.608	0.557	0	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
115	0.158	0.956	0.597	0.518	15.16*	0.012	0.120	2.222	0.523	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Number of SRUs: 3 **LRU DDR:** 1.0 **F:** 80.0 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.3 0.4 0.3
Repair Times: 5 for LRU; SRU$_i$ = 40 40 20
QPA of SRUs: 4 2 1

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
116	7.267	7.594	7.529	7.376	2.07*	0.143	18.858	18.858	10.588	10	12 8 4

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU
QPA of SRUs: 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
117	1.216	2.518	1.931	1.918	0.70	0.016	1.500	12.660	2.260	0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
118	0.520	0.698	0.618	0.690	− 10.46*	0.009	0.500	0.719	0.702	0	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
119	0.230	0.804	0.545	0.580	− 5.96*	0.010	0.019	3.493	0.723	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

TABLE 2-2

SIMULTANEOUS DETECTION CASES (Continued)

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 54.95 **Sum of SRU Failure Probabilities:** PSUM =

Conditional SRU Failure Probabilities: 0.5 for each SRU

Repair Times: 5 for LRU; 10 for each SRU

QPA of SRUs: 4 4 4 4 4 3 3 3 3 3 2 2 2 2 2 1 1 1 1 1

Case #	ALOW	AUP	EST	CAN	% ERR	DELTA	MLOW	MUP	NOCAN	Stock Levels	
										LRU	SRUs
120	0.158	0.515	0.354	0.371	− 4.54*	0.008	0.107	1.254	0.429	1	4 4 4 4 4 3 3 3 3 3 2 2 2 2 2 1 1 1 1 1

DISCUSSION OF RESULTS

The average absolute percent error from our technique – 3.57 percent – compares very favorably with the 306 percent error from METRIC. Our maximum error of –17.86 percent is a good deal larger than our 3.57 percent average. However, the METRIC error exceeds 100 percent in 40 of the 120 cases and was less than ours in only 7 cases.

One problem with average absolute percent error is that a few very large errors can distort the conclusion. Since the true value (CAN) is in the denominator of percent error, a few large percent errors can easily occur, especially when CAN is small. An alternative assessment using percent error can be obtained by summing the absolute errors and then dividing by the sum of the true values. This average error must be smaller, but the important question is how our error and the METRIC error compare. These errors of 2.31 percent and 93.43 percent, respectively, still show a ratio of approximately 1 to 40.

We also observe:

- Our errors are smallest for cases in which PSUM is less than or equal to one. This is true for METRIC as well, but as demand rates increase (e.g., Cases 21 and 37) the METRIC error also becomes very large.
- The error in 56 of our 120 cases falls inside the 95 percent confidence interval from the simulation
- Our largest percentage errors of –17.86 for Case 100 and –16.16 for Case 112 are small backorder errors of 0.009 and 0.005, respectively.
- When all SRU failure probabilities are one (Cases 88 through 100), the expected backorders without cannibalization, NOCAN, are identical with CAN. This is true for any repair distribution, and is due to the opportunistic replacement policy under NOCAN.
- In the other cases, NOCAN is usually only slightly larger than CAN. There are notable exceptions, however. In Case 37 NOCAN is 86 percent larger than CAN, in Case 104 NOCAN is almost twice as large, and in Case 112 NOCAN is more than twice as large. The similarity among these cases is the high LRU stock level and low SRU stock levels. The analytic upper bounds are below NOCAN in the first two of these cases. It is because of cases such as these and the fact that our analytic model is based on the cannibalization assumption that we chose to focus on the full canni-

balization case in this study. Note that in each of the 120 cases CAN lies between the bounds of ALOW and AUP.

- We have assumed Erlang-4 repair distributions for the SRUs. It may be of interest to assess the sensitivity of backorders to that assumption. The solution for constant SRU repair times lies between the lower analytic bound and CAN; that for exponential repair times lies between CAN and the upper analytic bound. For example, for Cases 88 through 90 the constant solutions are identical with the lower bound (because each SRU fails every time), and the exponential solutions are 1.841, 1.803, and 1.803, respectively.

CHAPTER 3

SEQUENTIAL FAILURE DETECTION

MATHEMATICAL MODEL

As noted in the *Introduction*, we assumed in the sequential failure detection case that after some LRU checkout time, a diagnosis is made of the first failed SRU. If a spare SRU is available, it is installed on the LRU and the testing continues to find the next (if any) failed SRU. However, if a spare is not available for the failed SRU, the diagnosis of the next failed SRU is delayed.

Again, we assume Erlang-4 repair times. In this sequential detection case, the mean and variance for the number of LRUs in repair are given by the VARI-METRIC approximations [3]:

$$E(x_o) = \lambda_o R_o + \sum_{J=1}^{N} E[B(s_J)] \qquad \text{[Eq. 3-1]}$$

$$Var(x_o) = \lambda_o R_o + \sum_{J=1}^{N} Var[B(s_J)] \qquad \text{[Eq. 3-2]}$$

where λ_0 is the LRU demand rate, assumed to be Poisson, and R_0 is the average LRU checkout time. Since any SRU with a backorder delays an LRU in this sequential case, it is appropriate to add the expected SRU backorders to obtain the total number of LRUs in repair.

However, the simulation output shows that Equations 3-1 and 3-2 tend to overstate the number of LRUs in repair when the SRU stock levels are positive, particularly when there are many SRU failures (e.g., 10 or so). The simulation shows that in cases with several SRUs in sequence and positive SRU stock levels, the probability distribution of the number in repair for the last SRUs in sequence is no longer Poisson. A degree of regularity has been imposed on the demand process such that the variance-to-mean ratios have dropped below the value of one for a Poisson to values in the 0.8 – 0.9 range. The positive SRU stocks act as a buffer in the demand process for the last SRUs in the sequence.

In the Appendix, we demonstrate procedures for calculating Erlang state probabilities for any specified variance-to-mean ratio less than one. If we knew the appropriate variance-to-mean ratio for each SRU, we could calculate the expected backorders and the variance more accurately (i.e., the last term in Equations 3-1 and 3-2, respectively). The problem is that the variance-to-mean ratio for a particular SRU is hard to predict – it depends on the stock levels of the previous SRUs in a rather complicated way.[3]

However, Equations 3-1 and 3-2 do give a good upper bound. We have found empirically that a reasonable lower bound is obtained by using Equations 3-1 and 3-2 with half of the SRUs. In cases in which the SRUs have different repair times, failure probabilities, or stock levels, we suggest that the SRUs for the lower bound be those with the largest SRU EBOs. That procedure was followed in the cases shown in Table 3-1.

As in the simultaneous failure detection cases, the difference between the lower and upper bound increases as the number of SRUs increases and as the sum of the SRU conditional probabilities, PSUM, increases. When PSUM is 10, the LRU EBOs tends to be about 67 percent of the distance from the lower bound to the upper bound. As PSUM decreases, the LRU EBOs are a larger percentage of the distance between the bounds. Using regression we found that the best fit was obtained with:

$$F = 1.126 - 0.196 \text{ Log(PSUM)}, \qquad \text{[Eq. 3-3]}$$

where F is the fraction of the difference between the lower and upper bound.

RESULTS

The numerical results of applying the approximation techniques described are shown in Table 3-1. The headings for each group of cases are identical with those in Table 2-2. The cases in this table were selected from the set of 120 cases in Table 2-2. We have used a much smaller number of cases here, concentrating on those with large values of PSUM where the differences are greatest.

[3]The LRU EBOs depend on the order in which the SRU failures are detected. An SRU with a small stock level will delay more LRUs if it is one of the first in the detection sequence.

Each line in the table provides the results for one case:

- *Case #:* Case number for reference; agrees with those in Table 2-2.
- *VLOW:* VARI-METRIC lower bound analytic solution for the LRU EBOs.
- *VUP:* VARI-METRIC upper bound analytic solution for the LRU EBOs.
- *EST:* Estimated solution for the LRU EBOs obtained by using the regression formula for the fraction of the distance between VLOW and VUP.
- *SIM:* Simulation result for the LRU EBOs under sequential SRU failure detection.
- *% ERR:* The percent error which is 100(EST-SIM)/SIM.
- *DELTA:* The delta value to be added and subtracted to SIM to obtain the 95 percent confidence interval for the LRU EBOs from the simulation. If EST falls outside the confidence limits for SIM, an asterisk is placed after the % ERR to indicate a statistically significant difference.
- *Stock Levels:* Stock levels for the LRU and each SRU.

The cases in Table 3-1 are presented in ascending order of PSUM, the sum of the SRU failure probabilities, except that the final cases, 116 through 120, have multiple applications (QPA) of some SRUs in the LRU.

By way of comparison with Table 2-2 for simultaneous detection, we should note that there is nothing like cannibalization in this table. If cannibalization were possible, the situation would be similar to simultaneous detection. Also the values of VLOW and VUP are similar to, but slightly larger than, MLOW and MUP in Table 2-2. The VARI-METRIC lower bound is set equal to the upper bound whenever PSUM is one or less and whenever all SRU stock levels are zero.

DISCUSSION OF RESULTS

The average absolute percent error from our technique – 6.30 percent – is much larger than the 3.57 percent obtained in Table 2-2. This is due in part to our concentration on cases with large values of PSUM and QPAs greater than one. However, the maximum percent errors are larger as well.

The comparable VARI-METRIC error, using the upper bound, is 9.62 percent. This is only 50 percent larger. However, the maximum error of 57.2 percent is reduced to 32.9 percent with our technique.

TABLE 3-1

SEQUENTIAL DETECTION CASES

Number of SRUs: 2 **LRU DDR:** 0.3 **F:** 112.6 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
14	0.419	0.419	0.419	0.420	− 0.24	0.012	1	1 1

Number of SRUs: 2 **LRU DDR:** 5 0 **F:** 112.6 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 2 for LRU; 5 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
21	10.082	10.082	10.082	9.934	1.49	0.283	25	0 0

Number of SRUs: 3 **LRU DDR:** 0.3 **F:** 112.6 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU_i Failure Probabilities: 0.6 0.3 0.1
Repair Times: 2 for LRU; 5 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
27	0.739	0.739	0.739	0.742	− 0.40	0.016	1	1 0 0
31	0.127	0.127	0.127	0.122	4.10	0.006	2	1 1 1

Number of SRUs: 3 **LRU DDR:** 1.0 **F:** 112.6 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU_i Failure Probabilities: 0.3 0.4 0.3
Repair Times: 5 for LRU; SRU_i = 40 40 20

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
36	6.812	6.812	6 812	6 945	− 1.92	0.202	10	12 8 4

TABLE 3-1

SEQUENTIAL DETECTION CASES (Continued)

Number of SRUs: 3 **LRU DDR:** 1.0 **F:** 102.2 **Sum of SRU Failure Probabilities:** PSUM = 5.0
Conditional SRU$_i$ Failure Probabilities: 0.6 0.8 0.3
Repair Times: 5 for LRU; 20 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
57	1.581	13.026	13.278	12.875	3.13*	0.259	25	0 1 0

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 81.06 **Sum of SRU Failure Probabilities:** PSUM = 5.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
64	0.135	0.350	0.309	0.332	− 6.85*	0.015	2	1 1 1 1 1 1 1 1 1 1
65	0.510	0.519	0.517	0.516	0.25	0.007	0	3 3 3 3 3 3 3 3 3 3

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 81.06 **Sum of SRU Failure Probabilities:** PSUM = 5.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 20 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
85	0.825	2.314	2.032	2.226	− 8.72*	0.074	2	1 1 1 1 1 1 1 1 1 1

TABLE 3-1

SEQUENTIAL DETECTION CASES (Continued)

Number of SRUs: 10 **LRU DDR:** 0.1 **F:** 67.47 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 1.0 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
91	10.500	10.500	10.500	10.534	− 0.32	0.122	0	0 0 0 0 0 0 0 0 0 0
92	9.500	9.500	9.500	9.534	− 0.36	0.121	1	0 0 0 0 0 0 0 0 0 0
93	8.500	8.500	8.500	8.535	− 0.41	0.119	2	0 0 0 0 0 0 0 0 0 0
94	4.504	6.340	5.743	6.294	− 8.76*	0.088	1	0 1 0 1 0 1 0 1 0 1
95	1.466	3.205	2.639	2.633	0.24	0.077	1	1 1 1 1 1 1 1 1 1 1
96	0.825	2.315	1.830	1.856	− 1.39	0.068	2	1 1 1 1 1 1 1 1 1 1
97	0.418	0.797	0.674	0.507	32.88*	0.022	1	2 2 2 2 2 2 2 2 2 2
98	0.617	0.733	0.695	0.609	14.16*	0.011	0	3 3 3 3 3 3 3 3 3 3

Number of SRUs: 10 **LRU DDR:** 1.0 **F:** 67.47 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 1.0 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
100	0.738	3.958	2.911	3.249	− 10.42*	0.226	15	10 10 10 10 10 10 10 10 10 10

TABLE 3-1

SEQUENTIAL DETECTION CASES (Continued)

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 67.47 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
101	10.500	10.500	10.500	10.439	0.58	0.127	0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
102	9.500	9.500	9.500	9.438	0.66	0.126	1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
103	8.500	8.500	8.500	8.438	0.73	0.125	2	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
104	7.502	7.502	7.502	7.443	0.79	0.121	3	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
105	5.500	5.519	5.513	5.494	0.34	0.086	0	0 3 0 3 0 3 0 3 0 3 0 3 0 3 0 3 0 3 0 3
106	0.519	0.539	0.532	0.531	0.28	0.008	0	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
107	5.500	5.567	5.545	5.514	0.57	0.089	1	0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
108	0.800	1.721	1.421	1.458	- 2.51	0.043	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
109	0.350	1.010	0.795	0.857	- 7.20*	0.035	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
110	0.188	0.281	0.251	0.238	5.36*	0.011	1	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
111	0.800	0.815	0.810	0.741	9.33*	0.020	1	3 3 3 3 3 3 3 3 3 3 1 1 1 1 1 1 1 1 1 1

Number of SRUs: 20 **LRU DDR:** 1.0 **F:** 67.47 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels	
							LRU	SRUs
112	1.477	7.863	5.786	7.077	-18.25*	0.524	15	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5

TABLE 3-1

SEQUENTIAL DETECTION CASES (Continued)

Number of SRUs: 3 **LRU DDR:** 1.0 **F:** 112.6 **Sum of SRU Failure Probabilities:** PSUM = 1.0
Conditional SRU_i Failure Probabilities: 0.3 0.4 0.3
Repair Times: 5 for LRU; SRU_i = 40 40 20
QPA of SRUs: 4 2 1

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels LRU	Stock Levels SRUs
116	18.859	18.859	18.859	20.894	- 9.74*	0.312	10	12 8 4

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 67.47 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU
QPA of SRUs: 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels LRU	Stock Levels SRUs
117	7.375	12.375	10.748	12.945	-16.97*	0.133	0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
118	1.032	1.052	1.045	0.831	25.81*	0.015	0	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
119	2.655	3.702	3.361	3.538	- 4.99*	0.059	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Number of SRUs: 20 **LRU DDR:** 0.1 **F:** 67.47 **Sum of SRU Failure Probabilities:** PSUM = 10.0
Conditional SRU Failure Probabilities: 0.5 for each SRU
Repair Times: 5 for LRU; 10 for each SRU
QPA of SRUs: 4 4 4 4 4 3 3 3 3 3 2 2 2 2 2 1 1 1 1 1

Case #	VLOW	VUP	EST	SIM	% ERR	DELTA	Stock Levels LRU	Stock Levels SRUs
120	1.092	2.174	1.822	1.513	20.42*	0.032	1	4 4 4 4 4 3 3 3 3 3 2 2 2 2 2 1 1 1 1 1

The following observations are also made:

- The error in 19 of our 35 cases falls inside the 95 percent confidence interval from the simulation.

- A number of our larger errors are in Cases 116 through 120, those with QPAs greater than one. Those cases are particularly complex because the SRU demand is compound Poisson with variance-to-mean ratios that decrease from one SRU to the next in the detection sequence.

- When the SRU stock levels are zero, the LRU EBOs are independent of the SRU repair distribution shape (Palm's theorem holds). When the SRU stock levels are positive, the LRU EBOs are largest when the SRU repair distributions are exponential and smallest when they are constant. For example, Case 97 has a large error and LRU EBOs of 0.507 under Erlang-4 repair. The backorders are 0.690 under exponential and 0.211 under constant repair. In the latter case, the variance-to-mean ratio for the number in repair of the last SRU is only 0.66.

- The simulation solution is sometimes below the lower bound (e.g., Cases 105, 111, and 118); in the cases with QPAs greater than one, the simulation solution is sometimes above the upper bound (e.g., Cases 116 and 117).

- A comparison of Tables 2-2 and 3-1 shows the expected backorders are always greater in the latter. The difference is smallest for low demand rates, ample stock, and small values of PSUM. The largest percentage difference is in Case 112 with values of 0.028 and 7.077 backorders in Tables 2-2 and 3-1, respectively. Other large differences are Cases 91, 101, and 117 for which Table 3-1 has values about five times as large as those in Table 2-2.

CHAPTER 4

CONCLUSIONS

We have demonstrated that the LRU EBOs are much larger in the sequential SRU failure detection case than in simultaneous detection. Obviously it is desirable for maintenance to strive for the latter type of detection process whenever possible. Simple approximate computational formulas that have been developed for both types of detection appear to give reasonably accurate results.

If a model such as VARI-METRIC is used without the suggested modifications, there are several implications. In the simultaneous detection cases, the LRU EBOs and the spares requirements will be overstated, dramatically in many cases. Thus, we would tend to buy too much stock although the proportion of budget spent on SRUs will tend to be about right.

In the sequential detection cases, the LRU EBOs and the spares requirements will be overstated but by a smaller amount. However, because VARI-METRIC gives the correct answers for zero SRU stock levels and overestimates backorders when SRUs are in stock, it underestimates the value of SRU stock. Thus, the proportion of budget spent on SRUs will tend to be smaller than optimal.

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APPENDIX

COMPUTATION OF ERLANG STATE PROBABILITIES

Since Erlang-4 repair distributions have been used in our modeling, we wanted to be able to compute Erlang state probabilities, i.e., the probability distribution for the number of events in any fixed time period where the time between events has an Erlang distribution. Those probabilities are necessary to model the sequential failure detection process more accurately. They are useful in other modeling applications as well, and we have not seen these methods described elsewhere.

We begin with the Erlang-4 and then extend the results to the general case. Consider a Poisson process and multiply the mean by four. The time between successive events is exponentially distributed, of course, and the time between every set of four events is Erlang-4. Our objective is to compute the state probabilities for these latter Erlang events.

To compute the state probabilities we must relate the Poisson probabilities for the original process to those of the Erlang. For example, suppose that we observe no events in the Poisson process; then there were no Erlang events. If we observe four Poisson process events, there must have been one Erlang event since every fourth event is Erlang.

The problem arises when we observe a number of Poisson events not precisely divisible by four. For example, if we observe one Poisson event, it may or may not be an Erlang event, depending on our counting origin. With a random origin, there is a probability 1/4 that it is an Erlang event. The general relationship for the Erlang-4 state probabilities with mean, M, $e(i|M)$, is:

$$e(0|M) = P(0|4M) + 0.75p(1|4M) + 0.5p(2|4M) + 0.25p(3|4M) \qquad \text{[Eq. A-1]}$$

$$\begin{aligned} e(i|M) = {} & 0.25p(4i-3|4M) + 0.5p(4i-2|4M) + 0.75p(4i-1|4M) \\ & + p(4i|4M) + 0.75p(4i+1|4M) + 0.5p(4i+2|4M) \\ & + 0.25p(4i+3|4M) \qquad i = 1, 2, \ldots \end{aligned} \qquad \text{[Eq. A-2]}$$

where the p's are Poisson probabilities with a mean of 4M. It is easy to verify that the e's sum to one, and thus comprise a valid probability distribution. It is also easy to check that the mean of the e's is M (when each equation for e(i) is multiplied by i and summed, the coefficient of each Poisson probability, p(n), is n/4). The Erlang variance is a simple function of the p's also, and it will always be less than the mean.

The result is a simple analytic computation for Erlang state probabilities from Poisson probabilities. While the physical model for the Erlang-k is based on the k^{th} exponential event where k is integral, the state probabilities can be calculated for nonintegral k as well. This allows us to model any variance-to-mean ratio less than one. The generalized version of Equations A-1 and A-2 for nonintegral k can be written, but the expression for the coefficients is very complicated. For computational purposes, it is more useful to provide the equations for the first three Erlang probabilities, where K is used to denote [k], the integer less than or equal to k:

$$e(0|M) = p(0|kM) + p(1|kM)(k-1)/k + p(2|kM)(k-2)/k \; \ldots + p(K|kM)(k-K)/k \qquad \text{[Eq. A-3]}$$

$$e(1|M) = p(1|kM)/k + p(2|kM)(2/k) \ldots + p(K|kM)(K/k) + p(K+1|kM)(2k-K-1)/k \ldots + p(2K|kM)(2k-2K)/k \qquad \text{[Eq. A-4]}$$

$$e(2|M) = p(K+1|kM)(1-k+K)/k \ldots + p(2K|kM)(2K-k)/k + p(2K+1|kM)(3k-2K-1)/k \ldots + p(3K|kM)(3k-3K)/k \qquad \text{[Eq. A-5]}$$

The general pattern can be inferred easily, noting that the numerators of the successive coefficients in an equation increase by 1 to the value K/k and then decrease by 1 (modulo k). Any probability p(x) is in one equation or two equations, depending on whether k is integral, and the sum of its coefficients is 1. Thus, the e's are a probability distribution since the p's are a probability distribution. When the Erlang probabilities e(i) in Equations A-3 through A-5 are multiplied by i and summed, each Poisson probability, p(x), has a coefficient x/k showing that the mean of the Erlang is M.